13-6 Conditional Probability Formulas

Common Core State Standards

S-CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. **Also S-CP.A.2, S-CP.A.3, S-CP.B.6**

MP 1, MP 2, MP 3, MP 4

Objective To understand and calculate conditional probabilities

Make a plan so that you account for all possibilities.

MATHEMATICAL PRACTICES

SOLVE IT! Getting Ready!

Suppose the probability of rain on Saturday is 40%. What is the probability that you clean the garage on Saturday?

In the Solve It, you may have calculated the probability that it rains and you clean the garage and the probability that it does not rain and you clean the garage and then added the probabilities together.

Essential Understanding You can find conditional probabilities using a formula.

In the previous lesson, you found probabilities using frequency tables. For example, in the relative frequency table at the right, the probability that a sales representative had an increase in sales, given that he attended the training seminar, is $\frac{0.48}{0.8} = 0.6$.

	Attended Seminar	Did not Attend Seminar	Totals
Increased Sales	0.48	0.02	0.5
No Increase in Sales	0.32	0.18	0.5
Totals	0.8	0.2	1

With respect to probability, this would be

$$P(\text{increased sales} \mid \text{attend seminar}) = \frac{P(\text{attend seminar and had increased sales})}{P(\text{attended seminar})}.$$

This suggests an algebraic formula for finding conditional probability.

take note

Conditional Probability Formula

For any two events A and B, the probability of B occurring, given that event A has occurred, is

$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$, where $P(A) \neq 0$.

Problem 1 Using Conditional Probabilities

Pharmaceutical Testing **In a study designed to test the effectiveness of a new drug, half of the volunteers received the drug. The other half of the volunteers received a placebo, a tablet or pill containing no medication. The probability of a volunteer receiving the drug and getting well was 45%. What is the probability of someone getting well, given that he receives the drug?**

Think

How can you find $P(A)$?
A is the event that represents the sample space. In this case, it is the volunteers that received the drug. Since this is half of the volunteers, $P(A)$ is $\frac{1}{2}$.

Step 1 Identify the probabilities.

$P(B \mid A) = P(\text{getting well, given taking the new drug})$

$P(A) = P(\text{taking the new drug}) = \frac{1}{2} = 0.5$

$P(A \text{ and } B) = P(\text{taking the new drug and getting well}) = 45\%, \text{ or } 0.45$

Step 2 Find $P(B \mid A)$.

$P(B \mid A) = \frac{0.45}{0.5} = 0.9, \text{ or } 90\%$ Use the conditional probability formula.

The probability of getting well if someone received the drug is 90%.

Got It? **1.** The probability of a volunteer receiving the placebo and having his or her health improve was 20%. What is the conditional probability of a volunteer's health improving, given that they received the placebo?

Conditional probabilities are usually not reversible. $P(A \mid B) \neq P(B \mid A)$.

Problem 2 Comparing Conditional Probabilities

Pets **In a survey of pet owners, 45% own a dog, 27% own a cat, and 12% own both a dog and a cat. What is the conditional probability that a dog owner also owns a cat? What is the conditional probability that a cat owner also owns a dog?**

Think

How are the sample spaces limited in each of the conditional probabilities?
In $P(\text{cat} \mid \text{dog})$, the sample space is limited to the probability that a pet owner owns a dog. Similarly, $P(\text{dog} \mid \text{cat})$, the sample space is the probability that an owner owns a cat.

$P(\text{cat} \mid \text{dog}) = \frac{P(\text{owns cat and dog})}{P(\text{owns dog})}$ Definition $P(\text{dog} \mid \text{cat}) = \frac{P(\text{owns cat and dog})}{P(\text{owns cat})}$

$= \frac{0.12}{0.45}$ Substitute. $= \frac{0.12}{0.27}$

$\approx 0.267, \text{ or } 26.7\%$ Simplify. $\approx 0.444, \text{ or } 44.4\%$

The conditional probability that a dog owner also owns a cat is about 26.7%.
The conditional probability that a cat owner also owns a dog is about 44.4%.

Got It? **2.** The same survey showed that 5% of the pet owners own a dog, a cat, and at least one other type of pet.

a. What is the conditional probability that a pet owner owns a cat and some other type of pet, given that they own a dog?

b. What is the conditional probability that a pet owner owns a dog and some other type of pet, given that they own a cat?

c. **Critical Thinking** Can you calculate the conditional probability of owning another pet for a pet owner owning a cat and no dogs? Explain.

Because $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

You can use this form of the conditional rule when you know the conditional probability. You can also combine conditional probabilities to find the probability of an event that can happen in more than one way.

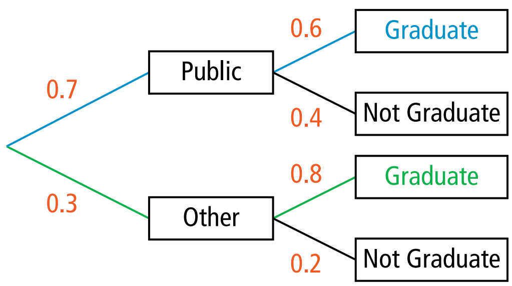
Problem 3 Using a Tree Diagram

Graduation Rate **A college reported the following based on their graduation data.**

- **70% of freshmen had attended public schools**
- **60% of freshmen who had attended public schools graduated within 5 years**
- **80% of other freshmen graduated within 5 years**

What percent of freshmen graduated within 5 years?

You can use a tree diagram to organize the information.



$$
\begin{aligned}
P(\text{Public and Graduate}) &= P(\text{Graduate, given Public}) \cdot P(\text{Public}) \\
&= 0.6 \cdot 0.7 \\
&= 0.42
\end{aligned}
$$

$$
\begin{aligned}
P(\text{Other and Graduate}) &= P(\text{Graduate, given Other}) \cdot P(\text{Other}) \\
&= 0.8 \cdot 0.3 \\
&= 0.24
\end{aligned}
$$

Plan

Why do you add the probabilities here?
You add the probabilities because the events are mutually exclusive.

$$
\begin{aligned}
P(\text{Graduate}) &= P(\text{Public and Graduate}) + P(\text{Other and Graduate}) \\
&= 0.42 + 0.24 \\
&= 0.66
\end{aligned}
$$

66% of the freshmen graduate within 5 years.

Got It? **3. a.** A soccer team wins 65% of its games on muddy fields and 30% of their games on dry fields. The probability of the field being muddy for their next game is 70%. What is the probability that the team will win their next game?

b. **Critical Thinking** If the probability of the field being muddy increases, how will that influence the probability of the soccer team winning their next game? Explain.

Lesson Check

Do you know HOW?

A jar contains 10 large red marbles, 4 small red marbles, 6 large blue marbles, and 5 small blue marbles. Calculate the following conditional probabilities for choosing a marble at random.

1. $P(\text{large} \mid \text{red})$

2. $P(\text{large} \mid \text{blue})$

3. For the same jar of marbles, which of the conditional probabilities is larger, $P(\text{red} \mid \text{small})$ or $P(\text{small} \mid \text{red})$? Explain.

Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. Error Analysis Your friend says that the conditional probability of one event is 0% if it is independent of another given event. Explain your friend's error.

5. Compare and Contrast How is finding a conditional probability like finding a compound probability? How is it different?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

See Problems 1 and 2.

6. Allowance Suppose that 62% of children are given a weekly allowance, and 38% of children do household chores to earn an allowance. What is the probability that a child does household chores, given that the child gets an allowance?

7. You roll two standard number cubes. What is the probability that the sum is even, given that one number cube shows a 2?

See Problem 3.

Softball **Suppose that your softball team has a 75% chance of making the playoffs. Your cross-town rivals have an 80% chance of making the playoffs. Teams that make the playoffs have a 25% chance of making the finals. Use this information to find the following probabilities.**

8. P(your team makes the playoffs and the finals)

9. P(cross-town rivals make the playoffs and the finals)

B Apply

10. Think About a Plan Suppose there are two stop lights between your home and school. On the many times you have taken this route, you have determined that 70% of the time you are stopped on the first light and 40% of the time you are stopped on both lights. If you are not stopped on the first light, there is a 50% chance you are stopped on the second light. What is the probability that you make it to school without having to stop at a stoplight?

- What conditional probability are you looking for?
- How can a tree diagram help?

11. **Sports** There is a 40% chance that a school's basketball team will make the playoffs this year. If they make the playoffs, there is a 15% chance that they will win the championship. There is also a 30% chance that the same school's volleyball team will make the playoffs this year. If the volleyball team makes the playoffs, there is a 30% chance that they will win the championship. What is the probability that at least one of these teams will win a championship this year?

STEM **Science** **In a research study, one third of the volunteers received drug A, one third received drug B, and one third received a placebo. Out of all the volunteers, 10% received drug A and got better, 8% received drug B and got better, and 12% received the placebo and got better.**

12. What is the conditional probability of a volunteer getting better if they were given drug A?

13. What is the conditional probability of a volunteer getting better if they were given drug B?

14. What is the conditional probability of a volunteer getting better if they were given the placebo?

STEM **15.** **Chemistry** A scientist discovered that a certain element was present in 35% of the samples she studied. In 15% of all samples, the element was found in a special compound. What is the probability that the compound is in a sample that contains the element?

16. **Music** Three students compared the music on their MP3 players. Find the probability that a randomly selected song is country, given that it is not on Student A's MP3 player. Round to the nearest hundredth.

Student	Rock	Country	R & B
A	0.10	0.02	0.03
B	0.13	0.05	0.23
C	0.10	0.01	0.33

17. **Fire Drill** A fire drill will begin at a randomly-chosen time between 8:30 A.M. and 3:30 P.M. You have a math test planned for 2:05 P.M. to 3:00 P.M. If the fire drill is in the afternoon, what is the probability that it will start during the test?

Challenge

18. The table below shows the average standardized test scores for a group of students. If 35% of the students are seniors, 40% are juniors, and 25% are freshman, what is the probability that a student chosen at random will have a score at least 125?

	Average Score < 125	Average Score between 125 and 145	Average Score > 145
Freshman	48%	37%	15%
Junior	36%	52%	12%
Senior	13%	69%	18%

Standardized Test Prep

SAT/ACT

19. Which of the following statements is true?

(A) A kite has two pairs of congruent angles.

(B) The measure of an inscribed angle equals the measure of the arc it intersects.

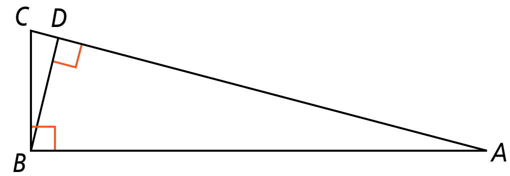
(C) A rhombus has two consecutive angles congruent if and only if the rhombus is a square.

(D) Two equilateral triangles can be combined to form a square.

20. If $CD = 3$ and $AD = 12$, what is BD?

(A) 2 (C) 6

(B) 4 (D) 8



Short Response

21. The sides of a rectangle are 5 cm and 12 cm long. What is the sum of the lengths of the rectangle's diagonals?

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 2

Review the information on page 823 about the survey Loretta's Ice Cream conducted and the question Loretta asked her assistant. To answer Loretta's question, her assistant needs to find these two probabilities:

- the probability that a customer who likes raspberry also likes caramel
- the probability that a customer who likes caramel also likes raspberry

a. The probabilities listed above are conditional probabilities. Write the conditional probability formula for the probability that a customer who likes raspberry also likes caramel.

b. What two probabilities do you need to find in order to use the conditional probablity formula to find the probablity that a customer who likes raspberry also likes caramel? Use the the information in the table you completed in the Apply What You've Learned in Lesson 13-1 to find these probabilties.

c. What is the probability that a customer who likes raspberry also likes caramel? Round your answer to the nearest whole percent.